What are the Computational Challenges for Cortex?

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Light Bulb Problem

Large *N*

Cognitive Tasks?

E.g. How many neurons do you use to remember each new person you meet at HLF7?

1, 10, 10^2, …. , or 10^7?

Symbolic Processing

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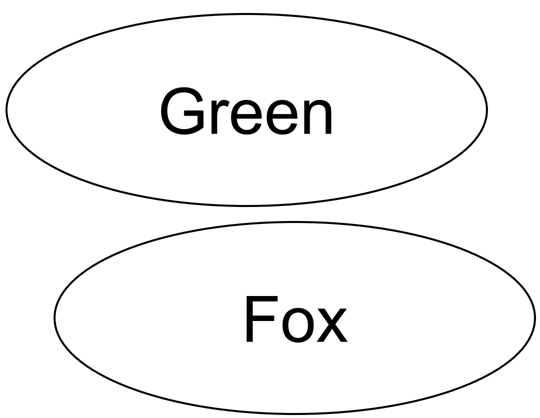

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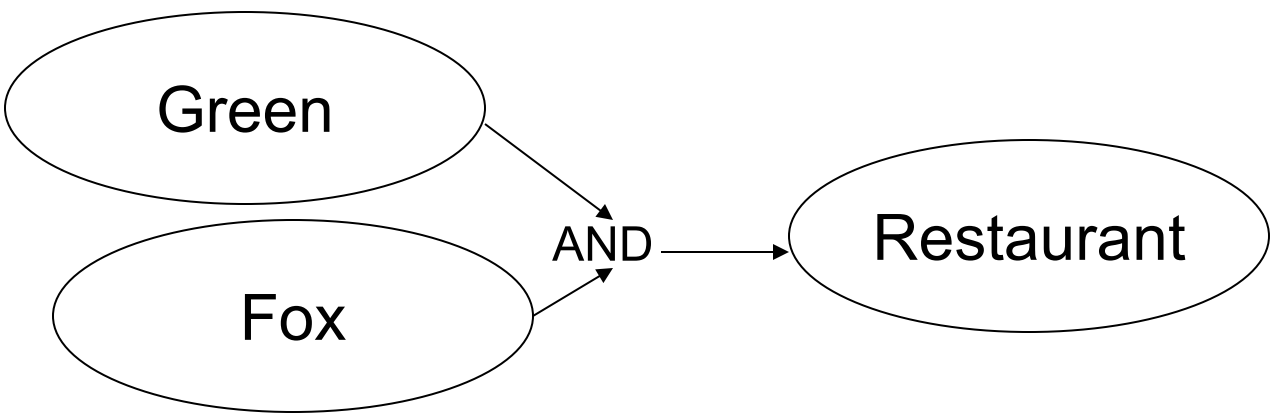

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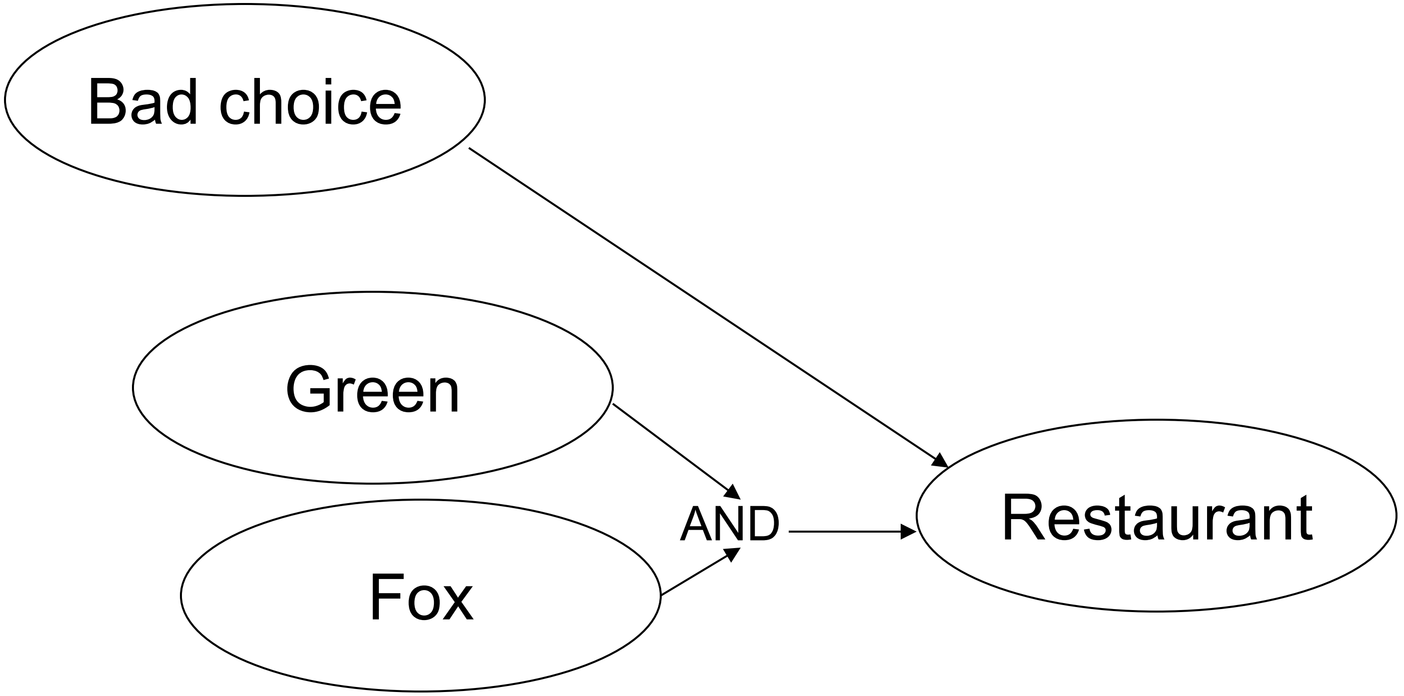

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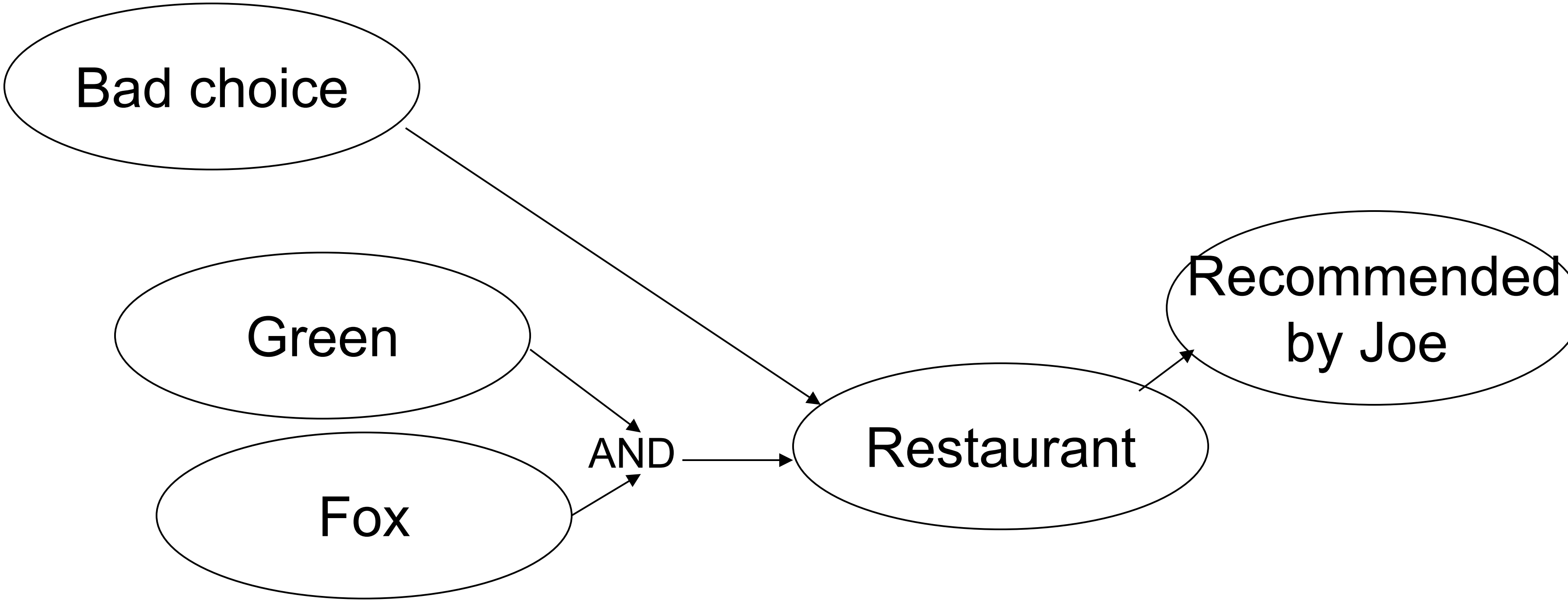

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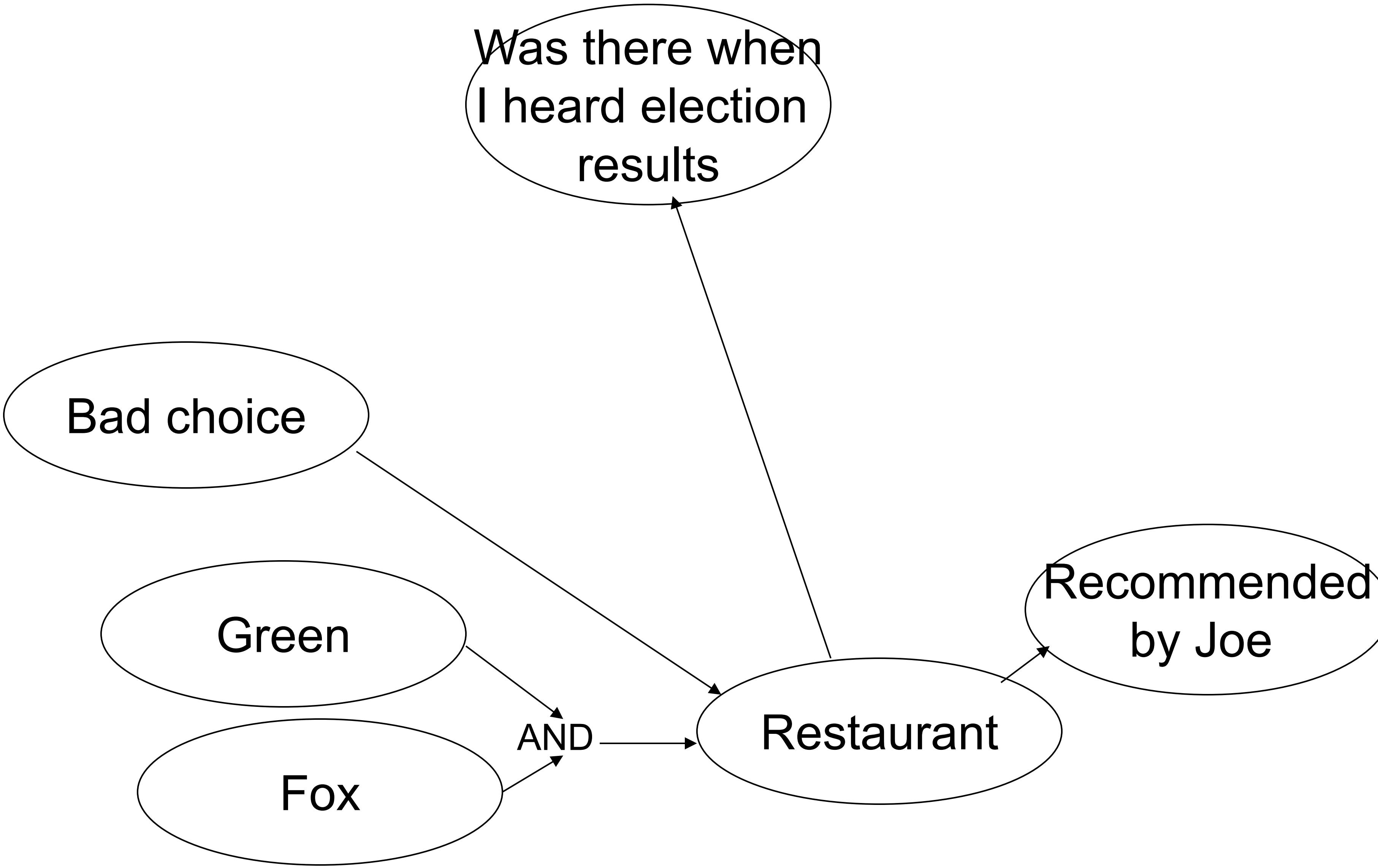

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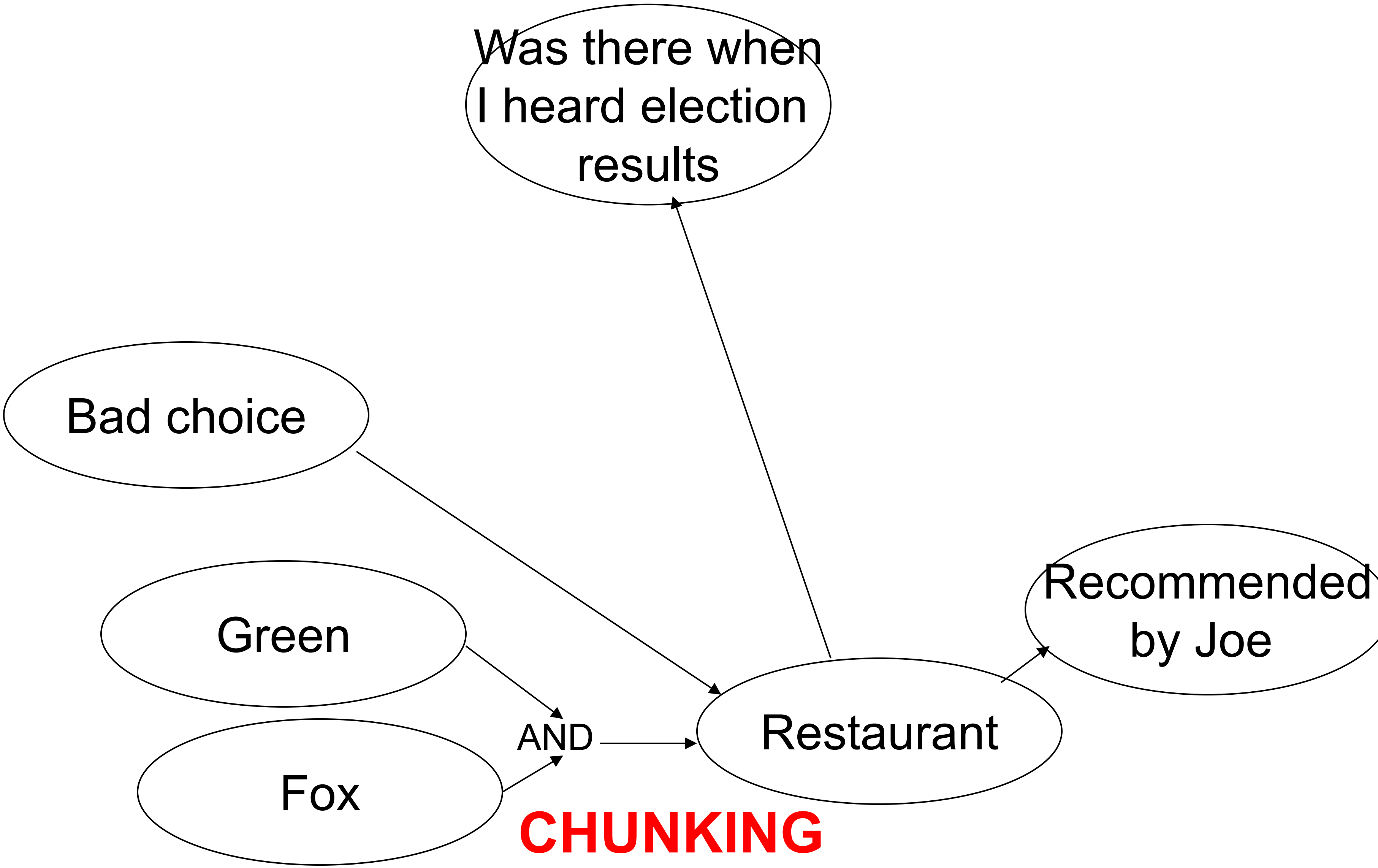

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No *generally agreed* theory known of how it can, even in principle.

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7. Cognitively adequate set of primitives

Constraints on Brain Computation

- No addressing mechanism!
- Slow – has to do much in 100 steps.
- Neurons sparsely connected, communication – challenged.
- Resource constraints:

  n neurons,

  d connections to/from each,

  maximum synaptic strength $1/k$.
- But long distance communication by stylized spikes – information carried in the timing.

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- (Aim to underestimate cortex.)

Neuroidal Model as Resource Model

- n neurons
- each connected to and from d others.
- max. synaptic weights $1/k \times$ threshold.
- time

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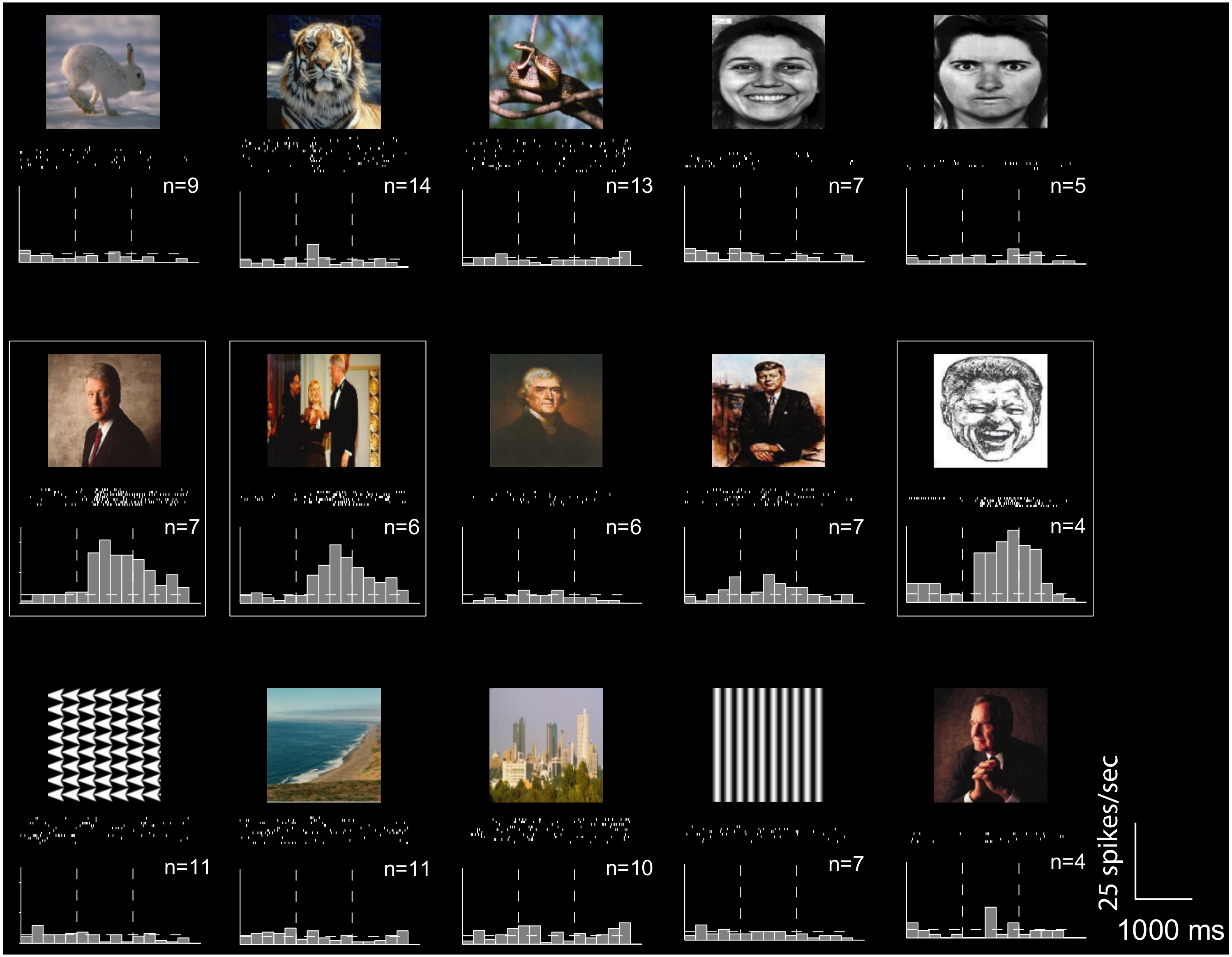
Note: Correspondences between items and neurons are "experimentally determinable"

r is large.

(e.g. in hippocampus, IT, olfactory bulb)



Kreiman, Fried, Koch (2002) *PNAS* **99**:8378; Crick, Koch, Kreiman, Fried (2004) *Neurosurgery* **55**:273

Are Sets Random?

The set of grid cells for any one offset looks random.

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Allocate Storage to New Concept

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*Constructs or modifies a **circuit** in response to stimulus.*

Random Access Tasks: Type II

Add relationships among represented concepts

(2) (e.g. Boris Johnson→ Prime Minister)

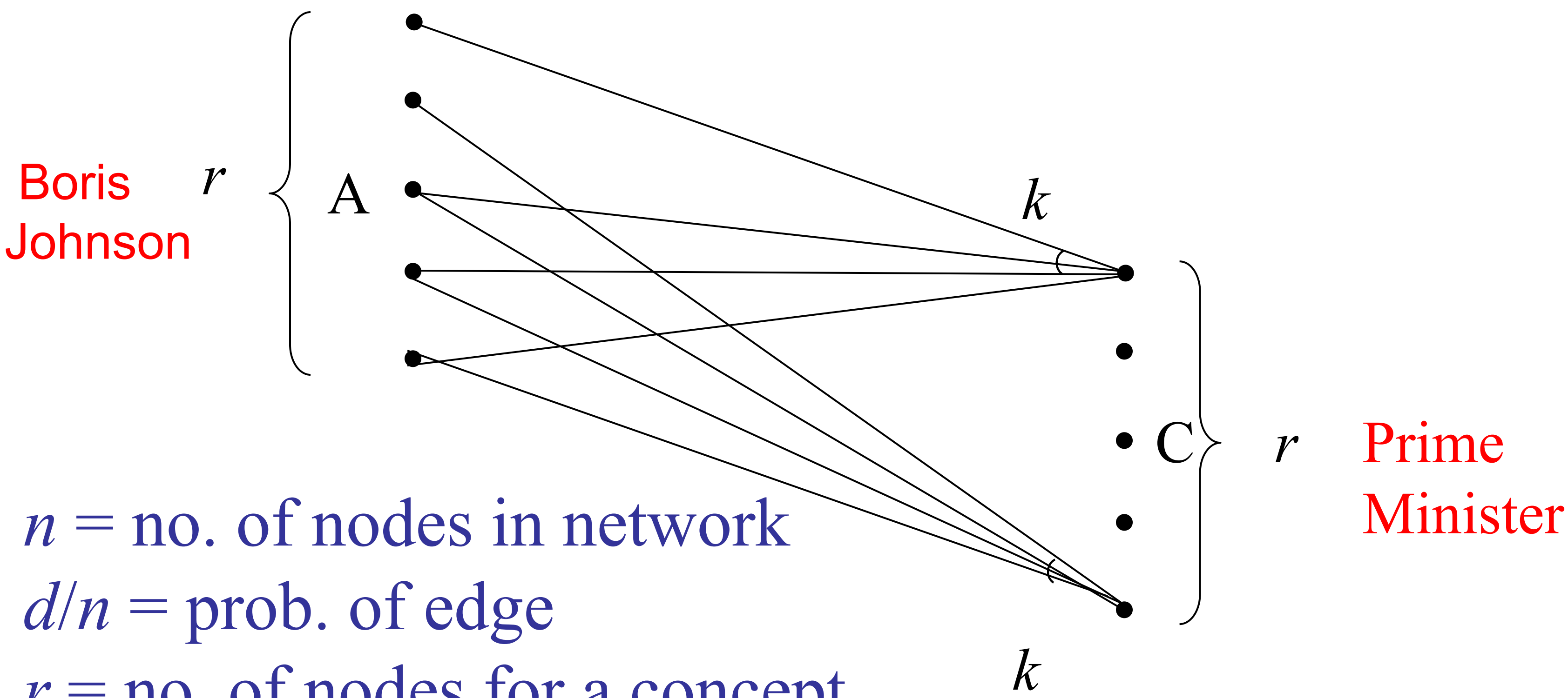
Association: For any stored items A, B, change synaptic weights so that in future when A is active then B will be caused to be also.

(c.f. Willshaw 1969)

Representations: Disjoint or Shared?

- Disjoint: Each neuron represents just one, possibly complex, item.
- Shared: Each neuron may represent many items.

Network Requirement for **Association** on Random Graph



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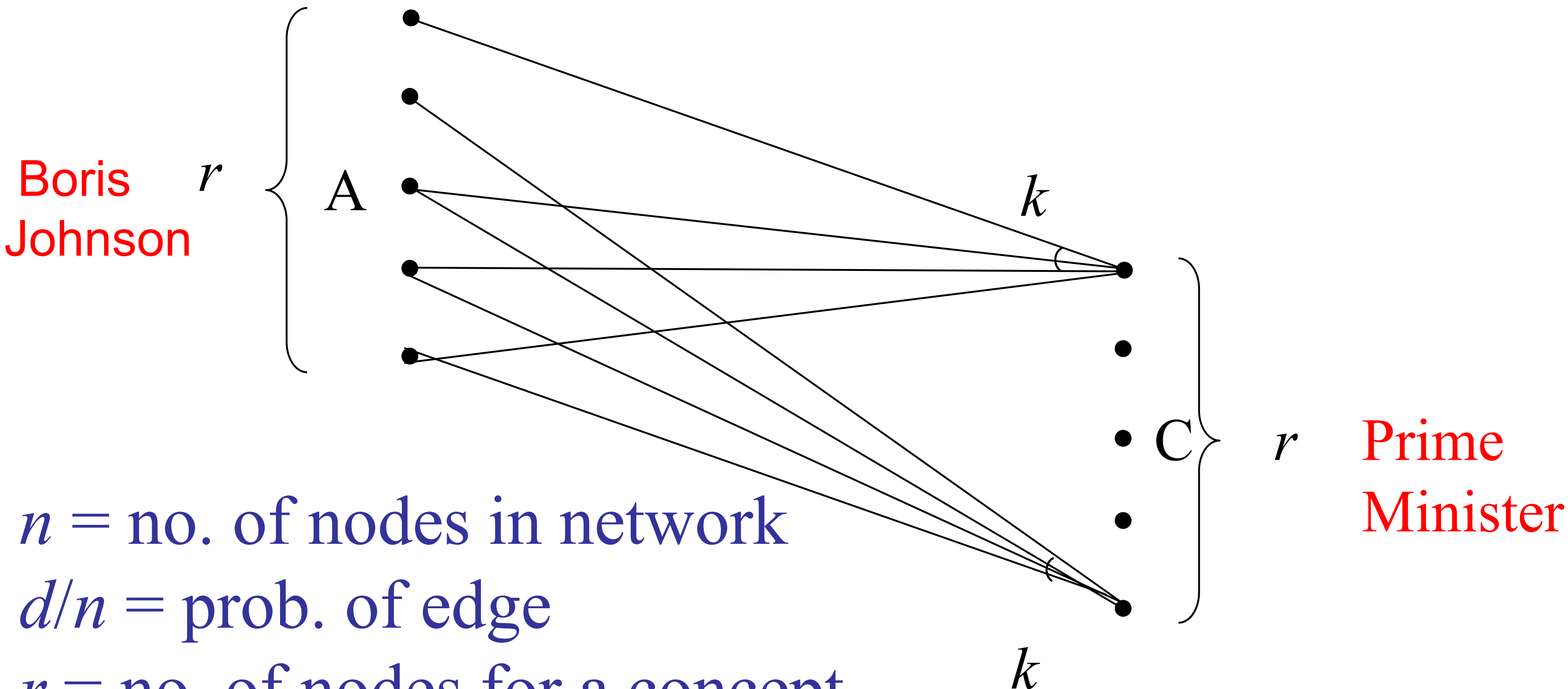

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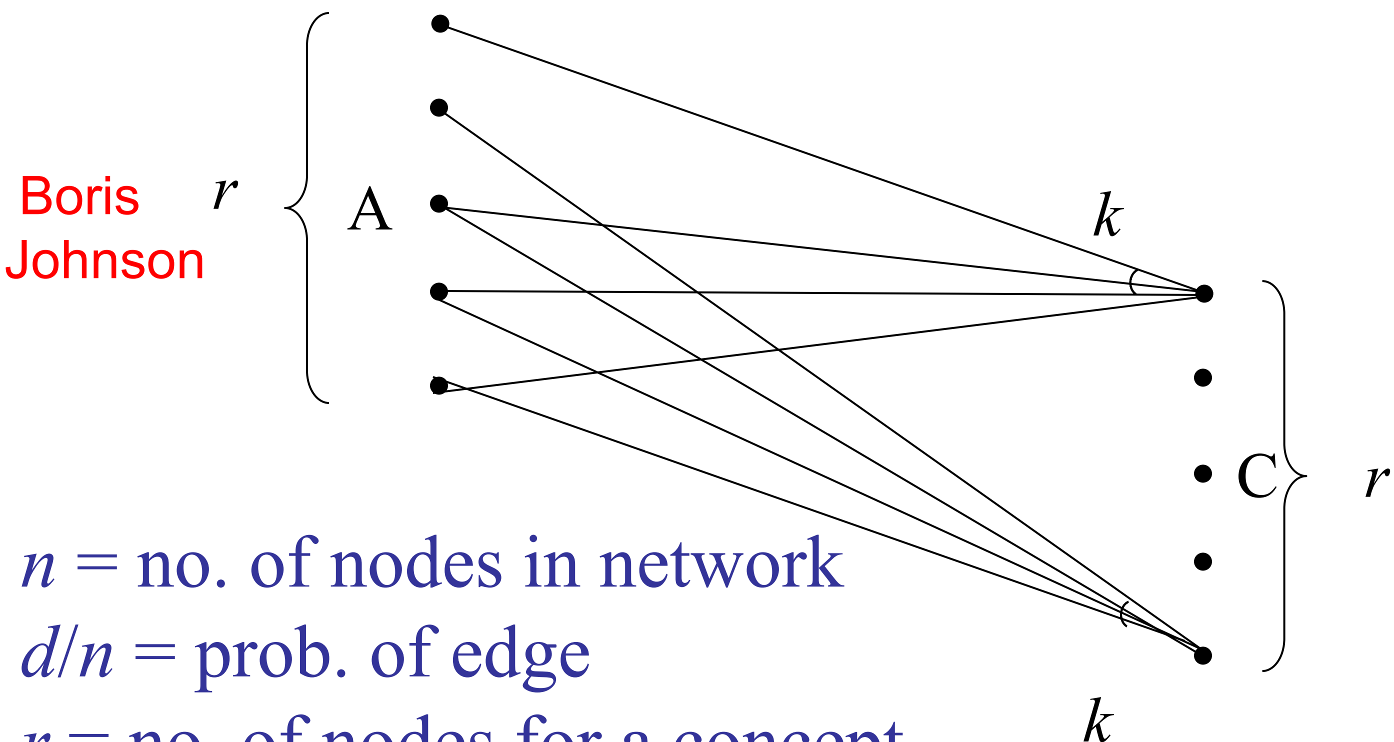

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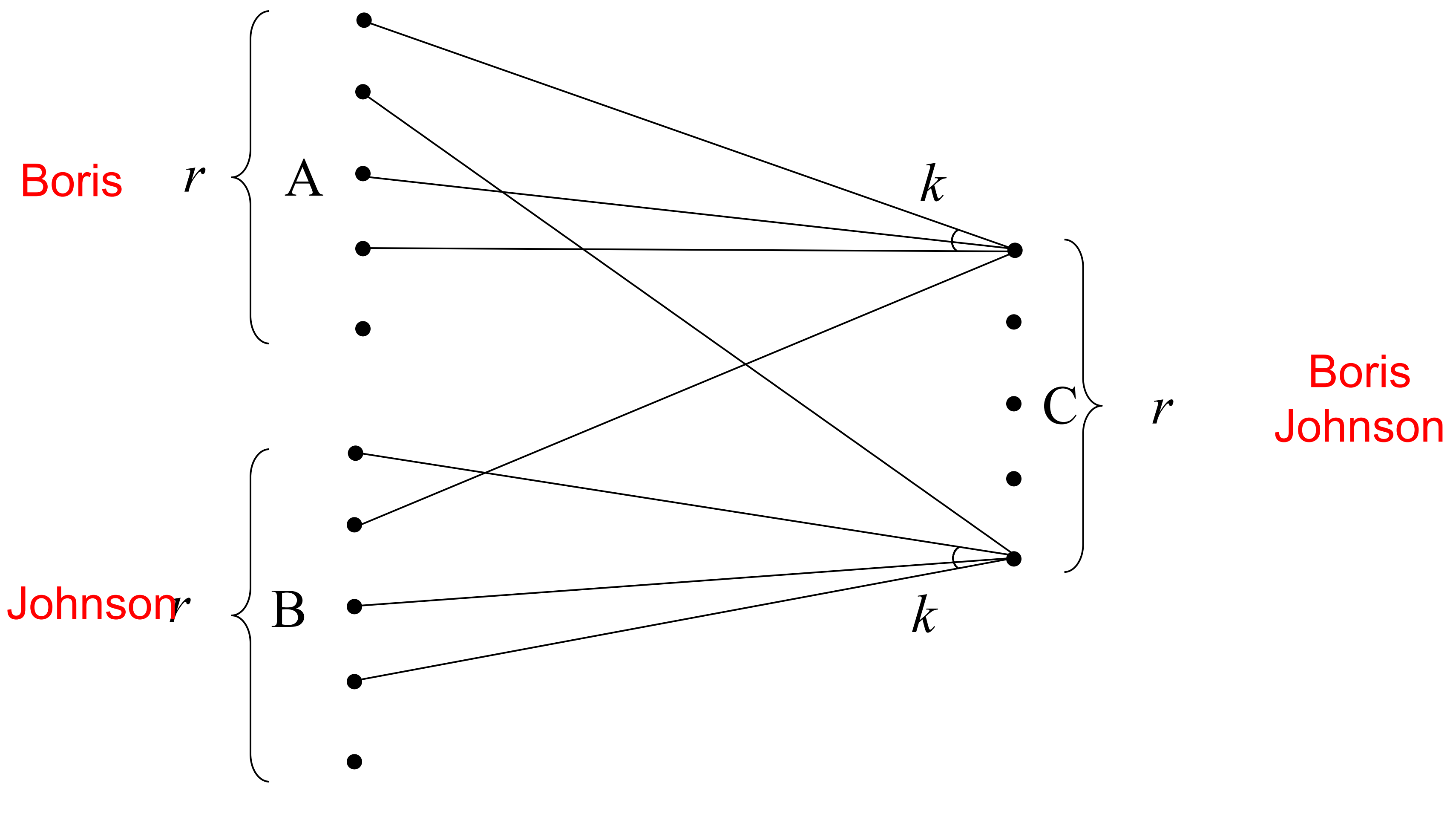
Association: a systems level Hebb's Rule?

Hierarchical Memorization



Actual Conditions Are More Complex

…..

Finding Capacity by Simulations

(V. Feldman & LV, *Neural Computation*, 2009)

Simulate mixed sequences of associations, supervised memorization, and inductive learning tasks, on initial allocation by hierarchical memorization.

Results of Simulations: Regime α

(V.Feldman & LV, *Neural Computation*, 2009)

$n = 10^8$ neurons.

$d = 8{,}000$ connections per neuron.

$k = 16$ (i.e. inputs from 16 needed for a.p.)

$r = 360000$ neurons per item, shared.

$\downarrow$

Sequences of 3,200 actions can be supported with small interference.

Results of Simulations: Regime β

(V.Feldman & LV, *Neural Computation*, 2009)

$n = 10^8$ neurons.

$d = 4{,}000$ connections per neuron.

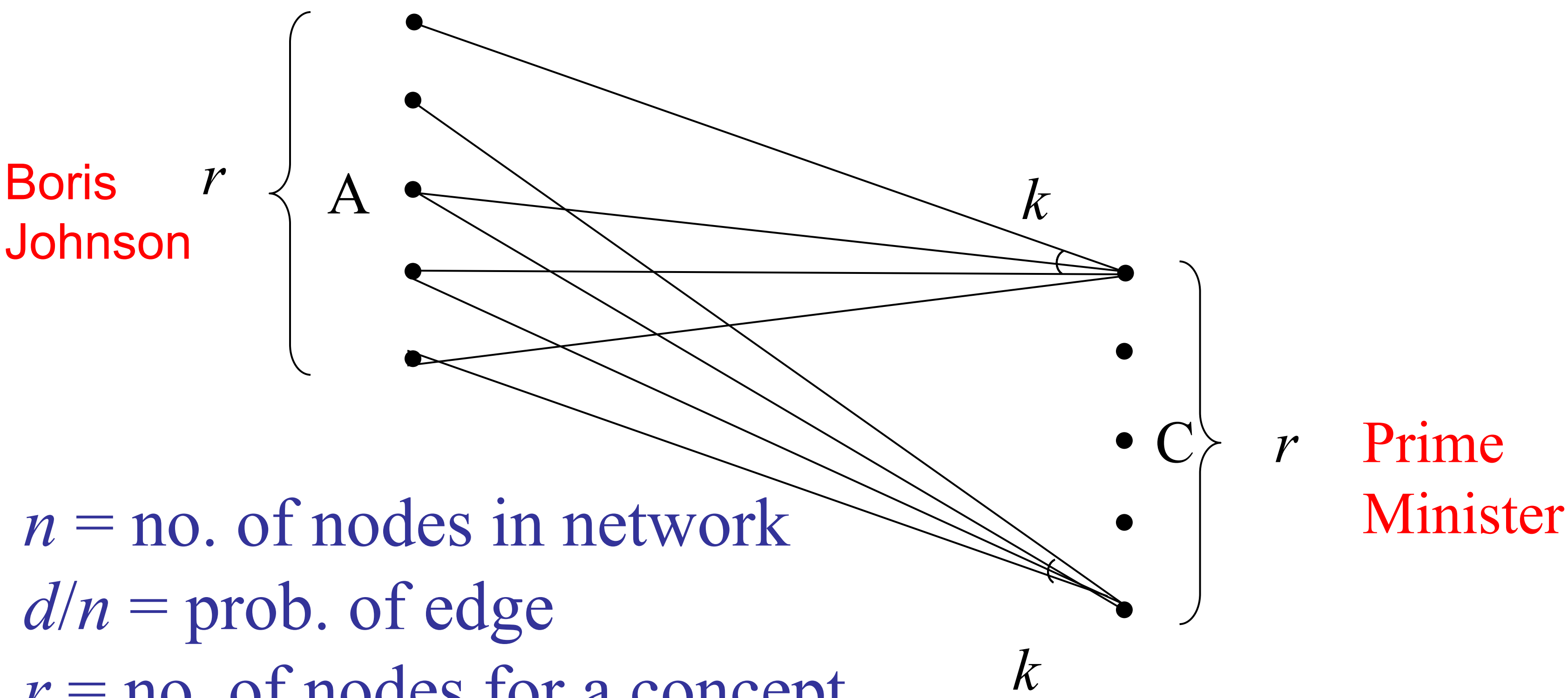
$k = 1$ (i.e. maximally strong synapses)

$r = 100$ neurons per item, disjoint.

↓

Sequences of 250,000 actions can be supported with small interference.

Network Requirement for **Basic Mechanism** for Association



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Complexity analysis for realizing Association

Association instance:

$(X_1 \rightarrow Y_1, \ldots, X_C \rightarrow Y_C)$: w.h.p. if X_i all fire Y_i all fire, and ~0 of rest.

How large can C be in terms of n, d, k?

In general $|X_i| = R$, $|Y_i| = r$.

For composability $R = r$.

Can $C \sim dn$ be achieved? E.g. Can $C \sim n^{3/2}$ if $d = n^{1/2}$?

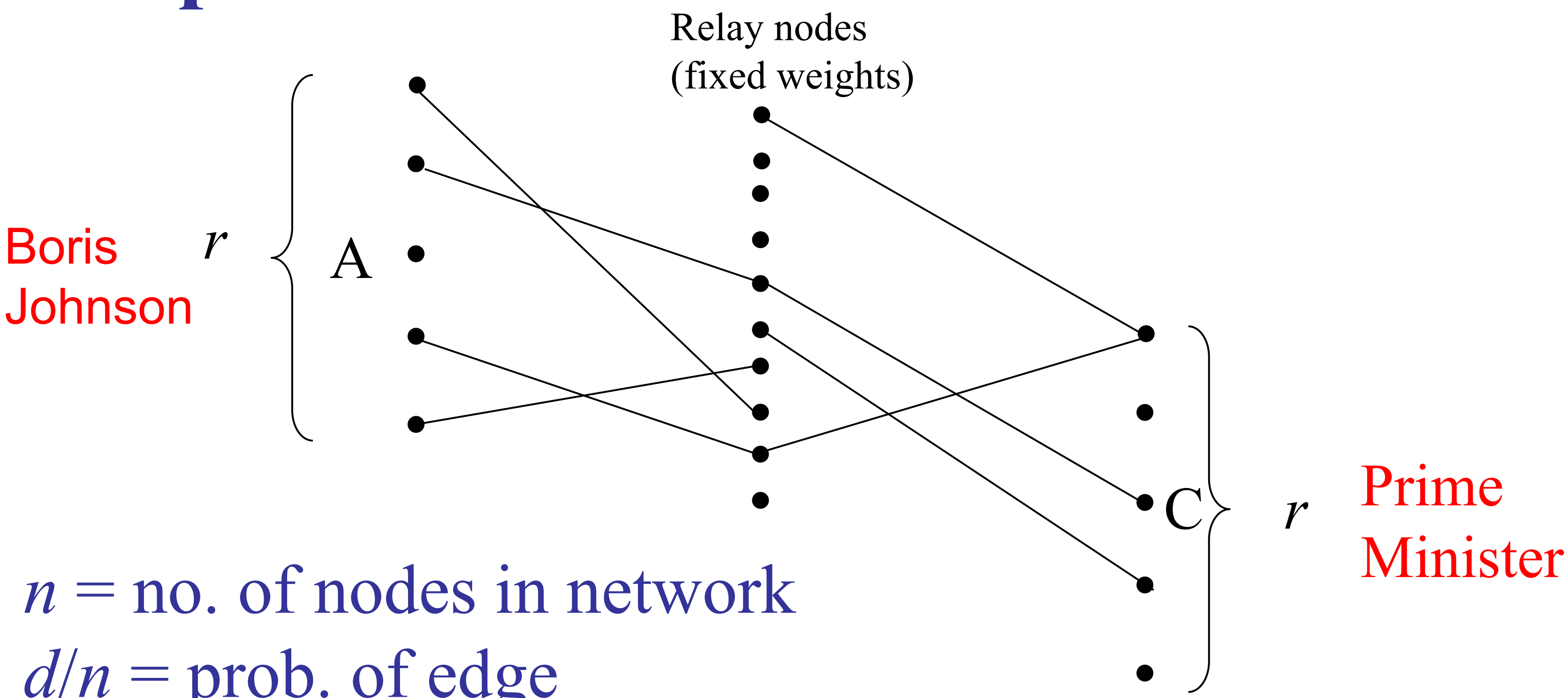
Theorem 1 *If … $C = \Theta(dn)$ achieved by* ***Basic Mechanism*** *to polylog with $R = nr/d$, $r = 3k$, $k = K\log_2 n$, for K large enough.*

Theorem 2 *If … for the* ***Basic Mechanism*** $C \leq (d^2R)/(k^2r)$.

($O(n)$ if $d = n^{1/2}$ and $r = R$).

(c.f. Willshaw 1969)

Network Requirement for **Expansive Mechanism** for Association



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Locust

How might these “systems level primitives” be validated experimentally?

“In-circuit” testing.

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Want to determine whether cortex is capable of this building block of computation.

THANK YOU